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Technical Notes

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Laminar Boundary Layers Subjected to High-Frequency Traveling-Wave Fluctuations

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Nomenclature

c_f = local skin friction coefficient
$F(x, y)$ = time-mean wave function
f' = dimensionless velocity, u/U
Q = traveling-wave convection velocity
Re_x = Reynolds number, Ux/ν
Re_θ = momentum thickness Reynolds number, $U\theta/\nu$
t = time
$U, \bar{U}$ = time-mean freestream velocity
U_e = unsteady freestream velocity
U_1 = wave amplitude
U_w = freestream wave velocity
u = velocity in the x direction
$\bar{u}$ = time-mean velocity in the x direction
u_w = wave component of velocity in the x direction
v = velocity in the y direction
$\bar{v}$ = time-mean velocity in the y direction
v_w = wave component of velocity in the y direction
x = streamwise distance
y = cross stream distance
α = momentum thickness coefficient, $\int_0^\infty f'(1-f')\,d\eta$
γ = traveling-wave convection velocity ratio, Q/U
Γ = wave parameter, $\varepsilon^2\sigma/\gamma$
δ = boundary-layer thickness, defined at $u = 0.99U$
δ_0 = oscillatory boundary-layer thickness, $\sqrt{2\nu/\omega}$
ε = freestream velocity amplitude factor, U_1/U
η = similarity coordinate, $y\sqrt{U/\nu x}$
θ = momentum thickness
ν = kinematic viscosity
σ = Strouhal number, $\omega x/U$
ω = frequency

Introduction

IN many aeronautical engineering problems, boundary layers are subjected to unsteady periodic freestream disturbances. These disturbances have two principal forms, namely, standing-wave disturbances, where the perturbation is everywhere in phase, and traveling-wave disturbances where the perturbation is convected along the boundary layer by the freestream. Here, attention is focused on the disturbances typically found in turbomachinery where the wakes of upstream blades are convected over the downstream blade row.[1] These wakes contain a turbulent component, typically generated in the combustor, and a traveling-wave component which results from the interaction of rotating blade rows and stationary vanes.[1,2] In this complex flow environment, engineers are concerned principally with the time-mean effects of these disturbances which are distinctive in that they occur at very high frequencies and may have large amplitudes. In this Note, the special case of laminar boundary layers subjected to high-frequency traveling-wave fluctuations is examined, where the disturbance amplitude may be arbitrarily large. The effect of turbulence has not been considered in this Note so that the traveling-wave effects can be understood and evaluated. The approach adopted here leads to a closed-form solution that yields useful engineering correlations based on a convenient dimensionless parameter. This parameter may be useful for the analysis of experimental data and the development of computational procedures.

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General Method

The method described here invokes the technique first described by Lin.[3] Consider the unsteady incompressible laminar boundary-layer equations for conservation of mass

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{1}$$

and momentum

$$\frac{\partial u}{\partial t} + u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = \frac{\partial U_e}{\partial t} + U_e\frac{\partial U_e}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} \tag{2}$$

subjected to the external unsteady freestream oscillation

$$U_e(x, t) = \bar{U}(x) + U_w(x, t) \tag{3}$$

and the usual no-slip boundary conditions at the wall. Correspondingly, the u and v components in Eqs. (1) and (2) can be separated into mean and wave components:

$$u(x, y, t) = \bar{u}(x, y) + u_w(x, y, t) \tag{4a}$$

$$v(x, y, t) = \bar{v}(x, y) + v_w(x, y, t) \tag{4b}$$

Substituting the relations (3) and (4) into Eqs. (1) and (2), time-averaging, and dropping the mean-flow quantity overbars for convenience, results in

$$\frac{\partial u}{\partial x} + \frac{\partial v}{\partial y} = 0 \tag{5}$$

for the mass equation, and

$$u\frac{\partial u}{\partial x} + v\frac{\partial u}{\partial y} = U\frac{\partial U}{\partial x} + \nu\frac{\partial^2 u}{\partial y^2} + F(x, y) \tag{6}$$

for the momentum equation where

$$F(x, y) = \overline{U_w\frac{\partial U_w}{\partial x}} - \left(\overline{u_w\frac{\partial u_w}{\partial x}} + \overline{v_w\frac{\partial u_w}{\partial y}}\right)$$

The equation governing the pure fluctuation of the wave can be derived by subtracting the time-averaged equations from the non-time-averaged equations. This results in a very ungainly nonlinear equation which readily simplifies when the condition $(\delta_0/\delta)^2 \ll 1$ is

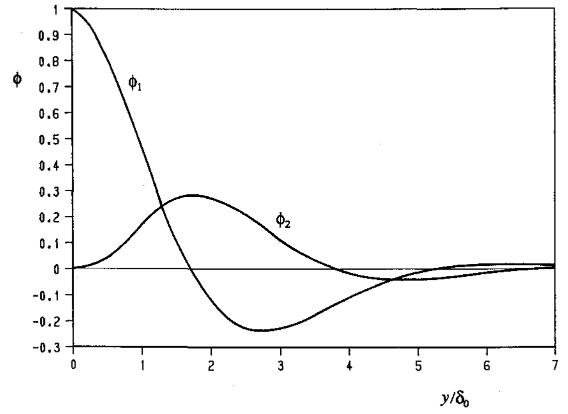

Fig. 1 Functions containing y/δ_0 terms in Eq. (8).

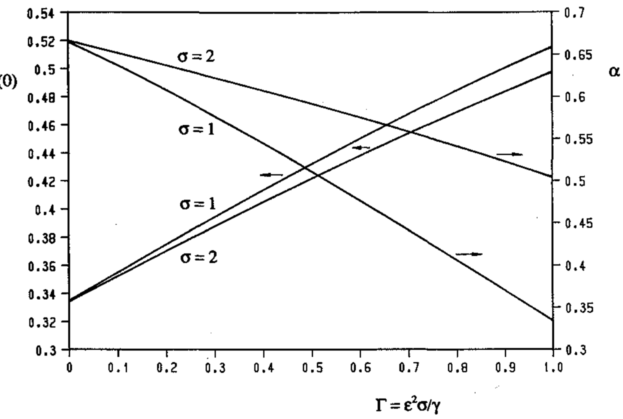

Fig. 2 Effect of the wave parameter on skin friction and momentum thickness.

satisfied, where $\delta_0 = \sqrt{2\nu/\omega}$. Thus, the mass and momentum equations governing the motion of the wave are

$$\frac{\partial u_w}{\partial x} + \frac{\partial v_w}{\partial y} = 0 \tag{7a}$$

and

$$\frac{\partial u_w}{\partial t} + \frac{\partial U_w}{\partial t} + \nu \frac{\partial^2 u_w}{\partial y^2} \tag{7b}$$

For the current investigation we have adopted Evans' characterization of the wave[4]:

$$U_w(x, t) = U_1(x) \sin \omega(t - x/Q)$$

Substituting this relation into Eq. (7b), together with the usual no-slip boundary conditions, yields an expression for u_w. Equation (7a) is then readily integrated for v_w and the form of $F(x, y)$ can be calculated:

$$F(x,y) = \frac{U_1}{2}\frac{\mathrm{d}U_1}{\mathrm{d}x}\Big[(2+y/\delta_0)\cos(y/\delta_0)$$

$$+ (y/\delta_0 - 1)\sin(y/\delta_0) - e^{-y/\delta_0}\Big]e^{-y/\delta_0}$$

$$+ \frac{U_1^2\omega}{2Q}\Big[(1 - y/\delta_0)\cos(y/\delta_0)$$

$$+ y/\delta_0 \sin(y/\delta_0) - e^{-y/\delta_0}\Big]e^{-y/\delta_0} \tag{8}$$

where the first term in the expression $0.5U_1 dU_1/dx\phi_1(y/\delta_0)$ is that derived by Lin and the second term $0.5U_1^2\omega/Q\phi_2(y/\delta_0)$ represents the traveling-wave contribution. These functions are plotted in Fig. 1. It is evident from the figure that the traveling-wave contribution has a markedly different form than the expression derived by Lin. The remainder of this Note is devoted to assessing the effect of this latter contribution.

Results and Evaluation

To illustrate the time-mean effect of the traveling wave on the laminar boundary layer, the classical flat-plate problem has been considered here. The external fluid stream is assumed to be free of pressure gradient and the amplitude and velocity of the traveling wave are assumed constant. For this problem, the outer edge of the boundary layer is defined at the point $u = 0.99U$, and consequently $(\delta_0/\delta)^2 = 2/25\sigma$. It can be seen by inspection that the condition $(\delta_0/\delta)^2 \ll 1$ holds for typical gas turbine blade Strouhal numbers with the exception of the region very close to the leading edge. Introducing the well-known Blasius substitutions,[3] the mass equation is automatically satisfied and the momentum equation reduces to the ordinary differential form:

$$ff'' + 2f''' + (\varepsilon^2\sigma/\gamma)\phi\sqrt{(\sigma/2\eta)} = 0 \tag{9}$$

subject to the boundary conditions

$$\eta = 0:\ f = 0;\ f' = 0, \quad \eta \to \infty:\ f' \to 1$$

where the prime denotes differentiation with respect to η. For convenience we shall define the wave parameter $\Gamma = \varepsilon^2\sigma/\gamma$. The aforementioned nonlinear ordinary differential equation was readily solved using a fourth-order Runge-Kutta integration scheme.[4]

Making use of the principle of similarity, the local skin-friction and momentum-thickness Reynolds number may be expressed as

$$c_f/2 = f''(0)/\sqrt{Re_x} \quad \text{and} \quad Re_\theta = \alpha\sqrt{Re_x}$$

respectively.[3] The quantities $f''(0)$ and α are plotted in Fig. 2 as a function of the wave parameter Γ, for σ values of 1 and 2. In general, the local skin-friction coefficient increases with the wave parameter whereas the momentum thickness decreases. All curves exhibit a very small negative second-derivative type curvature but can be adequately represented by linear functions for the range of Γ presented here. The skin friction increases by approximately 50% as the wave parameter increases from 0 to 1. This quantity also exhibits a very small dependence on σ with the difference between the two results typically around 3%. Using the Reynolds analogy, it is evident that traveling-wave fluctuations can have a pronounced effect on the local heat transfer coefficient within the laminar boundary layer of gas turbine blades.

In many boundary-layer codes currently used for turbine blade design, boundary-layer transition is triggered when the momentum-thickness Reynolds number reaches a critical value. It is evident from Fig. 2 that the momentum-thickness Reynolds number is strongly affected by both the wave parameter and the Strouhal number. For the condition $\sigma = 2$, α reduces by approximately one-quarter of its original value at maximum Γ, whereas for $\sigma = 1$ it reduces to half its value. The authors believe that this effect poses an added complication to the problem of predicting transition onset. Indeed, Mayle[5] points out that there is no design criterion for the prediction of wake-induced transition and also suggests that the local conditions within the laminar boundary layer should be related to transition onset. It should be noted, however, that although the traveling-wave phenomenon should be a consideration when analyzing transition, these results cannot be used in any way to predict transition onset.

Due consideration should also be given to the stability characteristics of these boundary layers as their mean profiles are markedly different to that of the Blasius profile. This feature is evident from Fig. 3 where profiles for $\Gamma = 1$ are compared with the Blasius profile. These curves qualitatively reflect the results of the previous figure.

In this Note, traveling-wave effects have been considered in the absence of a turbulent component. It is pointed out here that the effect of turbulence may be included with the traveling-wave, giv-

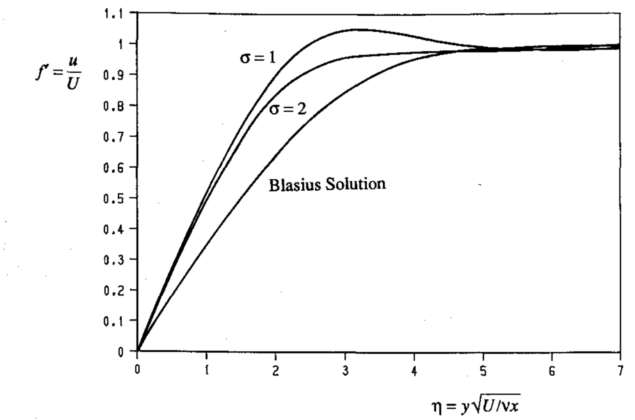

Fig. 3 Effect of the wave parameter and Strouhal number on time-mean velocity profiles.

ing rise to a computational procedure that is able to account for the effect of so-called "wake-passing" on the laminar boundary layer. A similar computational procedure, which is efficient and may be readily incorporated into existing boundary-layer codes, has been developed by Greenblatt[6] for standing-wave-type disturbances. In the light of recent attention that has been given to the effect of wake-passing on boundary layer transition,[5] a code that can predict the effect of wake-passing on the laminar boundary layer will compliment these developments. The authors believe that the work presented in this Note lays important elements of the foundation for such a code.

Conclusions

In this Note, the effect of high-frequency traveling-wave fluctuations on a flat-plate boundary layer was considered. The analysis yielded useful engineering correlations based on the dimensionless wave parameter $\Gamma(\equiv \varepsilon^2\sigma/\gamma)$ and the Strouhal number σ. It was shown that Γ has a pronounced effect on both the local skin-friction coefficient and the momentum thickness, whereas σ only affects the momentum thickness significantly. It was suggested that the traveling wave phenomenon should be considered when studying the complicated problem of transition onset.

Acknowledgment

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TURNS: A Free-Wake Euler/Navier-Stokes Numerical Method for Helicopter Rotors

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Introduction

THE accurate prediction of the aerodynamic flowfield, including the acoustics of a helicopter rotor, continues to be a complex and challenging problem despite the availability of modern supercomputers and improved numerical methods. This complexity stems from several peculiar features unique to helicopter rotors. Of these, the vortical wake plays a dominant role since it is largely responsible for unsteady load fluctuation, noise, and vibration. Also, it is the most difficult component of the flowfield to accurately predict.[1] Radiated noise can severely restrict rotorcraft usage in both civilian and military operations. Impulsive noise, the sum total of the blade-vortex interaction (BVI) noise and high-speed impulsive (HSI) noise, forms the most important component of the radiated noise. The BVI noise is generated due to the interaction of the vortical wake with the rotating blades and is generally more difficult to model due to the importance of unsteady, three-dimensional, and wake effects. HSI noise, on the other hand, is caused primarily by compressibility effects. If the advancing blade tip Mach number is highly supercritical, the phenomenon known as delocalization may occur, whereby the supersonic pocket on the rotor blade extends out to the farfield beyond the blade tip. If this occurs, the noise becomes more impulsive and, in particular, gets focused in its direction of propagation. Fortunately, the influence of lift on HSI noise appears to be secondary.[2] Thus, one can estimate this even with nonlifting configurations.

Recent studies using the Euler and Navier-Stokes methods have demonstrated the feasibility of using a single computational fluid dynamics (CFD) method to calculate simultaneously the aerodynamics and acoustics of a helicopter rotor in hover and forward flight.[3–5] This Note summarizes the computational capabilities of a numerical procedure, called TURNS[5] (transonic unsteady rotor Navier-Stokes), to calculate the aerodynamics and acoustics (HSI) out to several rotor diameters, all in one single solution without using any modeling either for wake or acoustic propagation. The vortical wake and its influence, as well as the acoustics, are captured as a part of the overall flowfield solution. Comparisons of the numerical results with the available experimental data demonstrate the accuracy and suitability of this method.

Numerical Method

The choice of govering equations affects the level of physics modeled and computational time. In this study, the unsteady Euler/Navier-Stokes equations are preferred to accurately model strong shocks and accompanying HSI noise, viscous-inviscid interaction, the flow in the tip region where the tip vortex forms, and the vortical wake. The Navier-Stokes equations are considered in the thin-layer approximation and an algebraic turbulence model[6] is used to obtain estimates of eddy viscosity for calculating turbulent flows. The govering equa-

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